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# DYNAMIC EQUILIBRIUM LIMIT-PRICING IN AN UNCERTAIN ENVIRONMENT

Garth Saloner*

November 1982
(Revised December 1984)

MIT Working Paper #342

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*Assistant Professor, Department of Economics, Massachusetts Institute of Technology. This paper was part of my Ph.D. dissertation. I would like to thank my advisor, John Roberts, for his encouragement and supervision. I have benefited from comments by Tim Bresnahan, Paul Milgrom, Robert Wilson, and seminar participants at Berkeley, Carnegie-Mellon, Columbia, Harvard, M.I.T., Northwestern, the University of Pennsylvania and Princeton.

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# DYNAMIC EQUILIBRIUM LIMIT-PRICING IN AN UNCERTAIN ENVIRONMENT

## 1. INTRODUCTION

In this paper we examine a multiperiod game-theoretic model of potential entry and limit pricing in the presence of exogenous demand uncertainty. Limit-pricing theory seeks to explain how manipulation by an established firm in the pre-entry periods can affect the entry decision of potential competitors by influencing the latters' perception of post-entry profitability. Most models of limit-pricing have utilized a maintained output hypothesis to provide the connection between low pre-entry prices and entry deterrence.[1]

Two additional and important themes have been developed in the literature. The first of these is the exploration of the inherent dynamics of the problem (e.g. Gaskins [6]). The second is the importance of uncertainty and of probablistic entry (e.g. Kamien and Schwartz [7], Baron [1]). In all this work the probability of entry by the potential entrant is typically exogenously specified to be an increasing convex function of current price. Limit pricing then emerges as the monopolist's optimal strategy and results in less entry than at the monopoly price.

However, these models suffer from a common defect noted by Friedman [3]: the form of the entrants' behavior is exogenously specified rather than being derived from rational decision making and, if one treats the entrants as decision-makers, then solutions based on the maintained output hypothesis fail the test of perfectness. A potential entrant with foresight will realize that it will not be sequentially rational for the established firms to maintain their pre-entry output if faced with *de facto* entry. Given this, under standard assumptions, pre-entry pricing will have no impact on the entry decision and so limit pricing would not occur.

In order to meet the perfectness issue head on and to endogenize the entrant's behavior, a number of game-theoretic models of entry have recently been developed in which all firms are modeled as rational players.[2] Of particular interest is the work of Milgrom and Roberts [10]. They note the fact that Friedman's perfectness argument against limit pricing rests on the presence of complete information about all decision-relevant parameters. Milgrom and Roberts then model the case of a monopolist facing a single potential entrant, with neither firm fully informed about the other's costs. If the entrant's profits are higher if it enters against a high-cost established firm than if it enters against a low-cost established firm, the monopolist may attempt to signal that it has a low cost structure in the hope of deterring entry.

This represents a valuable insight. However, it treats a very special case: there is no uncertainty in the environment other than about costs, and the dynamic structure is very simple. The current paper attempts to incorporate the important contributions of the dynamic, probablistic models in the game-theoretic framework. A model is examined in which not only is there incomplete information as to player types, but there is also exogenous demand uncertainty. If, as will be assumed, the established firm has to make its pre-entry qunatity choice before the demand uncertainty is resolved, and if price (but not output) is observable by the entrant, then the entrant can no longer exactly infer the type of the established firm from the observed price, even if its beliefs as to the output choices of each type of established firm are correct and these choices differ between types.

In a two period model it is shown that for a wide class of possible forms of entrants' beliefs and of the nature of stochastic demand, the optimal strategy for an entrant is to enter if and only if the observed price exceeds some "trigger price". Further, this trigger price is increasing in the entrant's (constant) average cost of production (or "type"). In equilibrium the established firm will produce an output in excess of the nomopoly output since this reduces the probability of the realized price exceeding any given trigger level and hence probabilistically deters entry. Also, since output exceeds the monopoly level, in expectation the price is below the monopoly price.

The basic structure of the model in the two period case is the same as that of a model developed independently by Matthews and Mirman [12]. Their paper analyses the case where the stochastic nature of the demand takes the form of an additive stochastic term with a density that has the entire real line as its support. So as not to be overly repetitive the current paper presents only those results in the two period model that are either essential for the development of the multiperiod model or are not presented in their paper. The interested reader is referred to their paper for an analysis of the amount of entry that occurs and for a worked numerical example.

In the Milgrom-Roberts model the potential entrant has only one opportunity to enter against the monopolist. Indeed only one entry opportunity is necesary in order to analyze all the implications of the model. Since the only information which is not known by the entrant is the established firm's cost structure, in a separating equilibrium observation of only one period's output choice is needed to provide the entrant with all the payoff-relevant information available.

With the introduction of exogenous noise the potential entrant at best receives a noisy signal of the monopolist's cost structure. In this context it then becomes important to examine a multiperiod model in which the potential entrant is able to receive multiple noisy signals and to form a more precise estimate over time of the monopolist's cost structure. A repeated game of this kind in which the players seek to maximize net present value is developed here. It is shown that if the entrant uses a trigger price strategy, the established firm will limit price in every pre-entry period. The entrant's problem is somewhat more complicated than in the two period model because in addition to deciding whether it is profitable to enter, the potential entrant also has to evaluate whether it is worthwhile delaying entry. The advantage to delaying entry arises from the fact that the longer the entrant stays out of the industry, the more price realizations it will observe. It is then able to use this additional information to make a more accurate assessment of the true costs of the established firm. On the other hand, the entrant gives up the expected profits it would earn from having entered. The entrant trades off these opposing factors in making its entry decision. An important implication of this model is that although entry may be delayed, if the time horizon is long enough there will eventually be more entry than with complete information. This arises because the long horizon implies that the entrant will ultimately have essentially complete information if it waits, while if he does not wait it will occasionally enter when it would not do so with perfect information.

## 2. A TWO PERIOD MODEL

This model treats the case of an established firm facing possible entry by a single potential entrant. The established firm is a monopolist during the first period. At the end of the first period the potential entrant has the choice of entering or remaining out of the industry. Thus each firm moves only once: the monopolist moves first choosing an output $Q_1$; and the potential entrant moves next choosing to enter or not.

The game is modeled as one of incomplete information on both sides. The incompleteness of information is, for simplicity, modeled as being with respect to the other firm's constant average cost of production. It could, however, be with respect to any payoff-relevant private information subject to some caveats presented below. The established firm has costs $c_1 \varepsilon [\underline{c}_1, \overline{c}_1] \subset R^+$ and the entrant's priors as to the established firm's costs are given by an atomless distribution with density $h_2(c_1)$. Similarly the entrant's costs lie in $[\underline{c}_2, \overline{c}_2] \subset R^+$ and the established firm's priors are $h_1(c_2)$. In general, the priors $h_j(c_k)$ could depend on the actual value of $c_j$;i.e. the firm's priors can be conditioned on its actual cost structure, but we will not consider this explicitly.

Exogenous uncertainty is introduced through the market inverse demand function. In particular, the realized first period price is given by $P_1 = P(Q_1, \tilde{\varepsilon})$. As usual we assume that $\partial P_1 / \partial Q_1 < 0$. Here $\tilde{\varepsilon}$ is a random variable. We assume that $\tilde{\varepsilon}$ is uniform on $[0,1]$; this involves no loss of generality, since, the conditional distribution of $P_1$ given $Q_1$ denoted $f(P_1 \mid Q_1)$ can be any continuous density. We denote the corresponding distribution function by $F(P_1 \mid Q_1)$. In particular, this general form of $P_1$ accommodates as a special case an additive noise term, $P_1 = P(Q_1) + \tilde{\varepsilon}$ or a multiplicative noise term, $P_1 =$

$P(Q_1)\tilde{\varepsilon}$. It is assumed that the function $f(P_1 \mid Q_1)$ is common knowledge. Further, the established firm learns the realized value of $P_1$ only after it has made its quantity choice, and the potential entrant observes only the realized price $P_1 = P(Q_1, \varepsilon')$, where $\varepsilon'$ is the realized value of the random variable $\tilde{\varepsilon}$ in period one.

With this description of the exogenous noise it is possible to describe exactly the informational structure of the game. At the beginning of the first period, when the established firm makes its quantity choice, it knows its own cost structure (type) $c_1'$; has priors $h_1(c_2)$ over the type of the potential entrant; knows its own cost structure of demand $P(Q_1, \tilde{\varepsilon})$ and $f(P_1 \mid Q_1)$ for all $Q^1$; but does not know what the realized value of $\tilde{\varepsilon}$ is going to be. At the end of the first period, when the potential entrant makes its enty decision, it knows its own cost structure (type) $c_2'$; its priors over the type of established firm, $h_2(c_1)$; the realized first period price $P_1' = P(Q_1, \varepsilon')$ and the general structure of demand, $P(Q_1, \tilde{\varepsilon})$ and $f(P_1 \mid Q_1)$ for all $Q_1$. It does not know the realization $\varepsilon'$, however, and hence is not informed about the established firm's choice of $Q_1$.

The first period payoff to the established firm is obviously independent of the potential entrant's entry decision. Thus in the first period the established firm earns $\pi^m(Q_1, c_1)$ which is the expected profit if it produces $Q_1$ when its costs are $c_1$ and it is the only firm in the industry. Both players are assumed to be risk neutral and hence payoffs can be expressed as their expected values without loss of generality. It is assumed that $\pi^m(Q_1, c_1)$ is strictly concave in $Q_1$ for a given $c_1$ and $\partial\pi^m(Q_1, c_1)/\partial c_1 < 0$. At the end of the first period the potential entrant makes its entry decision. If it decides to enter, the industry becomes a duopoly forever after, whereas if it remains out the established firm retains its monopoly forevever after and produces its monopoly output $Q_1^m(c_1)$. Thus the payoffs after the end of period one can be collapsed into a single figure for each firm, representing the discounted expected present value of the payoffs with the resulting duopoly structure:

TABLE 1

PAYOFFS FOR A GIVEN $(c_1', c_2')$ PAIR

| | MONOPOLY (entrant stays out) | DUOPOLY (entrant enters) |
|---|---|---|
| ESTABLISHED FIRM | $\pi^m(Q_1, c_1') + \pi^m(Q_1^m, c_1')$ | $\pi^m(Q_1, c_1') + \pi^1(c_1', c_2')$ |
| POTENTIAL ENTRANT | 0 | $\pi^2(c_1', c_2') - K$ |

As the table indicates, the established firm and potential entrant earn $\pi^1(c_1', c_2')$ and $\pi^2(c_1', c_2')$ respectively in duopoly. The entrant pays a fixed entry cost K if it decides to enter and its payoffs are normalized so that its payoff when it stays out of the industry is zero. When the potential entrant stays out the monopolist earns its monopoly profit forever, with an expected present value of $\pi^m(Q_1^m, c_1')$.

The payoffs under duopoly will depend on the behavior of the firms in the post-entry subgame. (For example, the payoffs could be thought of as being the result of the player's learning each others' cost structures and then playing Cournot. Or they could be the result of a quantity-setting game in which each firm has only its prior beliefs as to its opponent's costs updated by inferences made from the entry decision and the first-period price.) Rather than explicitly analyzing the post-entry subgame and specifying one particular solution, we instead require that the following conditions be satisfied: (i) $\pi^m(Q_1^m, c_1') > \pi^1(c_1', c_2')$ for all $(c_1', c_2')$; and (ii) $c_1'' < c_1' \rightarrow \pi^2(c_1'', c_2) < \pi^2(c_1', c_2)$ for all $c_2$. The first condition states that the established firm is strictly better off under monopoly than duopoly and the second condition requires that the entrant be better off against an established firm with a higher cost structure.

Given this simplified structure, the strategy sets of the two players are uncomplicated. A strategy for the established firm is a function $\sigma$ : $[\underline{c}_1, \bar{c}_1] \rightarrow [0, \bar{Q}]$ where $\bar{Q}$ is an upper bound on the output possibility set. Note that the output possibility set in conjunction with the specification of $P(Q_1, \tilde{\varepsilon})$ determines the supremum and infimum of the observable prices. We define these to be $\bar{P}_1$ and $\underline{P}_1$ respectively. Since the support of the probability density function $f(P_1 \mid Q_1)$ need not be bounded in general, $\bar{P}_1$ and $\underline{P}_1$ take values in the extended reals. For most reasonable applications we would expect $+\infty > \bar{P}_1 > \underline{P}_1 \geq 0$ and we assume this to be the case. The entrant observes not only its own cost, $c_2'$, but also the ex-post first period price $P_1'$. A strategy for the entrant, then, is represented by a function $\tau$: $[\underline{c}_2, \bar{c}_2] \times [\underline{P}_1, \bar{P}_1] \rightarrow \{0, 1\}$ where "0" denotes that the entrant stays out of the industry and "1" that it enters.

With perfect information a potential entrant with costs $c_2'$, would enter against an established firm with costs $c_1'$ if and only if its expected post-entry profits exceeded its entry costs; i.e. if and only if $\pi^2(c_1', c_2') \geq K$. Since $\pi^2(c_1', c_2')$ is increasing in $c_1$, the entrant may be willing to enter against existing firms of one type, but not against established firms with a lower cost structure. Since the problem is trivial if either all or none of the possible types of potential entrant would be prepared to enter indendent of the cost structure of the established firm, we suppose that either of these hold for at most a strict subset of the possible types of entrant, and that the union of these sets has measure less than the measure of $[\underline{c}_2,\overline{c}_2]$.

In the absence of complete information, the way in which the entrant interprets the realized prices will depend on its beliefs about the established firm's strategy. Define $\mu(c_1)$: $[\underline{c}_1,\overline{c}_1] \rightarrow [0,\overline{Q}]$ as the entrant's beliefs, or conjectures, about the output decision of each type of established firm. With a belief structure of $\mu(c_1)$, after the entrant has observed $P_1'$, it is able, through a process of Bayesian revision, to assess the probability that the established firm is of a particular type (given $P_1'$). It is thus able to calculate its expected profit on entry and decide whether or not to enter. Thus the belief structure $\mu(c_1)$ defines an optimal strategy $\tau^*$. This suggests the equilibrium concept to be used here, namely a fulfilled expectations equilbrium. An equilibrium is an expectation structure $\mu^*(c_1)$ and a pair of strategies $\sigma^*$ for the established firm and $\tau^*$ for the entrant such that:

(i) $\tau^*$ is optimal given $\mu(c_1)$;

(ii) $\sigma^*$ is a best response to $\tau^*$; and

(iii) $\mu^*(c_1) = \sigma^*(c_1)$ for all $c_1$.

In other words, the entrant behaves optimally given its beliefs, the established firm's strategy is a best response to that of the entrant, and in equilibrium the entrant's beliefs (or "expectations") are in fact correct. This is identical to the statement that $(\sigma^*, \tau^*)$ form a Nash equilibrium. The beliefs are introduced for expository purposes.

We limit attention to beliefs $\mu(c_1)$ that are decreasing in $c_1$, so that the entrant believes that low-cost established firms produce at least a much as higher-cost established firms. Then with no exogenous noise a relatively high price would be interpreted by the entrant as having been generated by a relatively high-cost established firm. In order to permit the entrant's inferences to have the same basic nature when noise is introduced, we assume that the family of densities $\{f(P_1 \mid Q_1)\}$ have likelihood ratios $f(P_1 \mid Q_1^*)$ which are monotone decreasing if $Q_1 > Q_1^*$ and increasing otherwise.[3] Then, as shown in Lemma 1, the entrant's expected profits will be monotonically increasing in the observed first period price, $P_1$. Notice that in general, since $\mu(c_1)$ need not be continuous, there may be prices $P_1 \, \varepsilon [\underline{P}_1, \overline{P}_1]$ to which the entrant assigns prior probability zero. For example, suppose that

$$\mu(c_1) = \begin{cases} Q_1' & \text{if } c_1 \varepsilon (c_1, c_1^*) \\ Q_1'' & \text{otherwise} \end{cases}$$

with $Q_1' > Q_1''$. Then the entrant would expect realized prices to be in the range of $P(Q_1', \tilde{\varepsilon})$ or $P(Q_1'', \tilde{\varepsilon})$. If $f(P_1 \mid Q_1')$ and $f(P_1 \mid Q_1'')$ have sufficiently small supports there will exist an intermediate range of prices that the entrant would never expect to observe. To rule out the possibility of gaps in the range of expected prices, we restrict attention to the class of expectations $\mu(c_1)$ and density functions for which the range of $P(\cdot)$ is a closed (connected) interval. This includes a wide class of situations. For example, it includes all cases where $\mu(c_1)$ is continuous; where $f(P_1 \mid \cdot)$ has unbounded support; or where although $\mu(c_1)$ is discontinuous and $f(P_1 \mid \cdot)$ is bounded, the support of $f(P_1 \mid \cdot)$ is sufficiently large to insure that there are no "gaps" in the range of $P(\cdot)$.

Notice that the elimination of "gaps" in the range of prices does not completely eliminate events with prior probability zero since prices greater than $\overline{P}_1$ or less than $\underline{P}_1$ are still possible. We make the further retriction on entrant beliefs that all prices greater than $\overline{P}_1$ are interpreted in the same way as a price of $\overline{P}_1$ by the entrant. Similarly for prices less than $\underline{P}_1$. This will allow the monotonicity which is shown to exist in the way the entrant interprets prices (Lemma 1) to be preserved. It should be pointed out, however, that the elimination of these events of prior probability zero in this way is not merely a technical convenience. Indeed it is at the root of the difference between this and the Milgrom-Roberts model for example. It provides for a continuity not otherwise present and results in a tighter characterization of equilibrium.

LEMMA 1: Let the potential entrant's expectations $\mu(c_1)$ be decreasing in $c_1$. Let the probability density functions $\{f(P_1 \mid Q_1)\}$ have likelihood ratios $f(P_1 \mid Q_1)/f(P_1 \mid Q_1')$ that are monotone in $P_1$; decreasing if $Q_1 > Q_1'$ and increasing otherwise. Then for all $c_2$, $E[\pi^2(c_1,c_2) \mid P_1'] \geq E[\pi^2(c_1,c_2) \mid P_1'']$ if $P_1' > P_1''$.

PROOF: Since $\{f(P_1 \mid Q_1)\}$ have the strict monotone likelihood ratio property,

$$f(P_1' \mid Q_1')f(P_1'' \mid Q_1) - f(P_1' \mid Q_1)f(P_1'' \mid Q_1') > 0$$

for all $Q_1 > Q_1'$. Let $c_1' \varepsilon \mu^{-1}(Q_1')$ and $c_1 \varepsilon \mu^{-1}(Q_1)$. Then since $Q_1' < Q_1$ and $\mu(c_1)$ is decreasing in $c_1$, the entrant assesses $Q_1'(c_1') \leq Q_1(c_1)$ if $c_1' \geq c_1$. Thus $f(P_1' \mid c_1')f(P_1'' \mid c_1) - f(P_1' \mid c_1)f(P_1'' \mid c_1') \geq 0$ for all $c_1' > c_1$, i.e., $\{f(P_1 \mid c_1)\}$ have the monotone likelihood ratio property. Since $\pi^2(c_1,c_2)$ is strictly decreasing in $c_1$ the result follows immediately. Q.E.D.

With this structure, the entrant believes that a high price is more likely to have been generated by a high-cost established firm than a low-cost firm. Thus its optimal strategy will be to enter if and only if the observed price exceeds some "trigger price". Since it is conceivable that the entrant's type may be such that it would always (never) want to enter since the profit it makes against the lowest (highest) cost type of established firm is always nonnegative (nonpositive), the trigger price may be $\underline{P}$ $(\overline{P})$.

LEMMA 2: The entrant's optimal strategy is of the form

$$\tau(c_2) = \begin{cases} 0 & \text{if } P_1 < P_1^*(c_2) \\ 1 & \text{otherwise} \end{cases}$$

for some $P_1^*(c_2)$.

PROOF: Clearly if $E[\pi^2(c_1,c_2') \mid P_1'] - K < 0$ for all $P_1$ then $P_1^*(c_2') = \overline{P}$. Similarly if $E[\pi^2(c_1,c_2') \mid P_1'] - K > 0$ for all $P_1$ then $P_1^*(c_2') = \underline{P}$. Now consider a value of $c_2'$ for which there exist $P_1'$, $P_1''$ such that $E[\pi^2(c_1,c_2') \mid P_1'] - K < 0$ and $E[\pi^2(c_1,c_2') \mid P_1''] - K \geq 0$. Since (i)$\pi^2(c_1,c_2)$ is continuous in $c_1$ and $c_2$, (ii) $f(P_1 \mid Q_1)$ is atomless, (iii) $c_1$ and $c_2$ are drawn from a continuum of types, and (iv) the range of expected values of $P_1$ is an interval, we have that $E[\pi^2(c_1,c_2) \mid P_1] - K$ is continuous in $P_1$. Therefore since $F[\pi^2(c_1,c_2') \mid P_1] - K$ is monotone in $P_1$(Lemma 1), $E[\pi^2(c_1,c_2') \mid P_1^0] - K = 0$ for some $P_1^0 \varepsilon [\underline{P}_1,\overline{P}_1]$. If $E[\pi^2(c_1,c_2') \mid P_1] - K$ is weakly monotone, the set of $P_1^0$ for which $E[\pi^2(c_1,c_2') \mid P_1^0] - K = 0$ may be an interval. Therefore set $P_1^*(c_2)$ equal to the infimum of that set. Clearly entering if and only if $P_1 \geq P_1^*(c_2)$ will ensure that the entrant enters if and only if expected profit is nonnegative.

Q.E.D.

The objective function of the established firm when the entrant's strategy is of this form is:

$$(1) \qquad \max_{Q_1} \pi^0(Q_1, c_1) = \pi^m(Q_1, c_1)$$

$$+ \int_{\underline{c}_2}^{\overline{c}_2} [\pi^1(c_1,c_2')(1-F(P_1^*(c_2') \mid Q_1)) + \pi^m(Q_1^m,c_1)F(P_1^*(c_2' \mid Q_1))]h_1(c_2')dc_2'.$$

This represents the expected first period profit, $\pi^m(Q_1,c_1)$, plus the expected second period profit. The latter is composed of post-entry duopoly profits multiplied by the probability of entry plus post-entry monopoly profits multiplied by the probability that no entry occurs. Equation (1) is assumed to be differentiable for a unique maximum. It can be rewritten as:

$$(2) \qquad \max \pi^m(Q_1,c_1) - \int_{\underline{c}_2}^{\overline{c}_2} [\pi^m(Q_1^m,c_1) - \pi^1(c_1,c_2')](1-F(P_1^*(c_2') \mid Q_1))h_1(c_2')dc_2'.$$

PROPOSITION 1: If $\mu(c_1)$ is strictly decreasing then every type of established firm limit prices (i.e., sets $Q_1 > Q_1^m(c_1)$) in equilibrium.

PROOF: If $\tau^*$ is an equilibrium strategy when $\mu^*(c_1)$ is nonincreasing in $c_1$, then it is a trigger strategy by Lemma 2. Then if $\sigma^*$ is a best response to $\tau^*$ it involves a choice of $Q_1$ for each $c_1'$ that maximizes $\pi^0(Q_1,c_1)$i.e.

$$\sigma^*(c_1) = \underset{Q_1}{\operatorname{argmax}}\ \pi^0(Q_1,c_1).$$

By differentiation of Equation (2) for a maximum:

$$\frac{\partial\pi^0(Q_1,c_1)}{\partial Q_1} = \frac{\partial\pi^m(Q_1,c_1)}{\partial Q_1}$$

$$- \int_{\underline{c}_2}^{\overline{c}_2} [\pi^m(c_1) - \pi^1(c_1, c_2')]f(P_1^*(c_2') \mid Q_1^*) \frac{\partial P_1(\cdot)}{\partial Q_1} h_1(c_2')dc_2' = 0.$$

Evaluating the derivative of the first period profit at the optimum $Q_1^*$:

$$\left.\frac{\partial\pi^{m}(Q_1,c_1)}{\partial Q_1}\right|_{Q_1=Q_1^{*}} = \int_{\underline{c}_2}^{\bar{c}_2} [\pi^{m}(c_1)-\pi^{1}(c_1,c_2')f](P_1^{*}(c_2') \mid Q_1^{*})h_1(c_2')dc_2' \left.\frac{\partial P_1(\cdot)}{\partial Q_1}\right|_{Q_1 = Q_1^{*}}$$

The quantity $[\pi^{m}(c_1) - \pi^{1}(c_1,c_2')]$ represents the established firm's reward to deterring an entrant of type $c_2'$ and is strictly positive by assumption. The term $\partial P_1(\cdot)/\partial P_1$ is strictly negative by the assumption of downward sloping demand. Therefore, the entire expression is negative (zero) if the measure of the set $\{c_2': f(P^{*}(c_2') \mid Q_1^{*}) > 0\}$ is positive (zero). It is shown in the appendix (Lemma 3) that this set has strictly positive measure. Therefore

$$\left.\frac{\partial\pi^{m}(Q_1,c_1)}{\partial Q_1}\right|_{Q_1 = Q_1^{*}} < 0$$

which implies that the established firm exceeds its monopoly output since $\pi^{m}(Q_1,c_1)$ is strictly concave. Q.E.D.

The set $\{c_2': f(P^{*}(c_2') \mid Q_1^{*}) > 0\}$ is precisely the set of potential entrants that may be drawn into the industry when the established firm produces $Q_1$. If this set is nonempty, (more precisely, if it has positive measure), when the established firm produces its monopoly output, it will have incentive to increase output beyond the monopoly level in order to reduce the probability that the realized price will exceed any given level, and thereby trigger entry. This result contrasts with a similar result in [10] where the limit pricing occurs almost everywhere rather than everywhere. Without the presence of noise the established firm with the highest cost structure will have no incentive to limit price in a separating equilibrium since no entry will be deterred by such an action. With noise, however, even this type of established firm benefits from increasing its output since this increases the probability that it will be mistaken for a lower cost type of established firm through a low realization of

$\tilde{\varepsilon}$. The notion that high prices in an industry are somehow indicative of high profits and hence would attract entry is ubiquitous in the industrial organization literature.[4] This model illustrates how this notion can be supported in a situation in which all the players behave as rational economic agents and where the price provides noisy information about post-entry profitability.

The dynamic models of limit-pricing referred to earlier assume that the probability of entry is an increasing function of price. Clearly this emerges endogenously here. The higher the price the greater is the probability that it exceeds any given trigger price and hence the higher is the probability of entry. Thus this long-standing idea is supportable in an equilibrium model.

In an equilibrium in which $\mu^*(\cdot)$ is strictly monotone, in expectation the entrant correctly infers from the price what the established firm's true costs are. In such a separating equilibrium entry occurs (in expectation) under the same circumstances as with perfect information. Because of the stochastic nature of prices, however, even in equilibrium there will be some error in the inferences made by the potential entrant. Thus a higher-than-average realization of $\tilde{\varepsilon}$ will lead some entrants to infer incorrectly that the established firm has a higher cost structure than it actually does. Thus with positive probability an entrant will regret that it chose to enter. On the other hand, balancing this, lower than average realizations of $\tilde{\varepsilon}$ will have the effect of deterring some potential entrants that would have entered if they had had perfect information.

The presence of noise thus makes price the relevant signal of profitability and at the same time clouds the inferences made by the potential entrant. As will be shown below, it also allows a more precise characterization of the equilibrium than was previously available.

Proposition 1 was derived assuming that an equilibrium (in which $\mu(c_1)$ is strictly monotone) exists. No proof of existence has been provided. Indeed not much is known about existence of equilibrium in games of incomplete information when there is exogenous uncertainty present. Most of what is known is contained in Milgrom and Weber [17]. However, even the assumptions that have been made ruling out the possibility of gaps in the range of expected prices do not provide enough continuity to use Theorem 2 of that paper to prove existence here. (If all types of potential entrant used the same trigger price, the incumbent's payoffs will be discontinuous in $\varepsilon$). Preliminary work by the author suggests that in some games of this kind pure strategy equilibria exist if there is "enough" noise, in the sense of a large variance. A complete characterization of the conditions under which existence will be guaranteed in the presence of exogenous uncertainty is beyond the scope of this paper. However, Matthews and Mirman [12] and Saloner [18] provide examples in which not only do pure strategy equilibria exist, but they are completely characterized. In the remainder of this section we show that certain other classes of equilibria can be eliminated. In the following section, as above, we characterize the features that an equilibrium has if it exists. The main justification for this approach, absent a proof of existence, is that worked examples do exist.

In [10] Milgrom and Roberts posit the possibility of the existence of an equilibrium in which $\mu^*(c_1)$ is essentially a step function. Such an equilibrium represents "partial pooling". In such an equilibrium, except possibly for an initial segment $[\underline{c}_1, c_1^*)$, the types of established firm are partitioned into disjoint connected subsets, with all the types of established firm in a given subset producing the same output. Proposition 2 shows that the existence of noise eliminates the class of step-function equilibria.

PROPOSITION 2: If $\mu^*(c_1)$ is a nonincreasing equilibrium belief structure, then $\mu^*(c_1)$ is continuous in $c_1$.

PROOF: Since $\mu^*(c_1)$ is nonincreasing in $c_1$, the optimal strategy for an entrant of type $c_2$ is a trigger strategy $P^*(c_2)$ by Lemma 2. When faced with trigger strategies sufficient continuity assumptions have been made to ensure that $\pi^0(Q_1,c_1)$ is continuous in $Q_1$. Then by a well-known theorem (Berge [2] pg 116) the mapping $Q_1^*$ defined by

$$Q_1^* = \{Q_1 \mid \pi^0(Q_1,c_1) = \max_{Q_1} \pi^0(Q_1,c_1)\}$$

is an upper hemicontinuous mapping of $[\underline{c}_1,\overline{c}_1]$ into $[0,\overline{Q}]$. But $\pi^0(Q_1,c_1)$ has a unique maximizing $Q_1$ for each $c_1 \varepsilon [\underline{c}_1,\overline{c}_1]$. Hence $Q_1^*(c_1)$ is continuous. Since $\mu^*(c_1) = Q_1^*(c_1)$ in equilibrium, $\mu^*(c_1)$ is continuous in $c_1$. Q.E.D.

Proposition 2 does not rule out the possibility that all types of established firm produce the same output, i.e., that $\mu^*(c_1) = x$ for some $x \, \varepsilon [0,\overline{Q}]$. Such an outcome would constitute complete pooling. Proposition 3 illustrates that if the noise has "sufficiently little" weight in the tails of the distribution, then no pooling equilibria can exist. The intuition for this is straightforward. Suppose that the entrant was expecting pooling. Then any price in the range $[\underline{P}_1,\overline{P}_1]$ of anticipated prices would result in the same amount of entry. Additional entry would be precipitated only by prices outside this range. If the noise puts very small weight in the tails then a deviation from the pooled output provides a first-order increase in expected profits to the established firm in the pre-entry period but results only in a small increase in expected entry.

PROPOSITION 3: If for all $x \geq Q_1^m(\underline{c}_1)$

$$1 - F(\overline{P}_1 \mid y) < \frac{\pi^m(y,\ c_1') - \pi^m(x, c_1')}{\pi^m(Q_1^m(c_1'), c_1') - \pi^1(c_1', \underline{c}_2)}$$

for some $y < Q_1^m(\underline{c}_1)$ and some $c_1' \varepsilon [\underline{c}_1, \overline{c}_1]$, then no complete pooling equilibrium exists.

PROOF: The formal proof is contained in the appendix. However, the condition is straightforward. $\pi^m(y, c_1') - \pi^m(x, c_1')$ represents the first period gain to the established firm for deviating below $Q_1^m(\underline{c}_1)$ to $y$. $\pi^m(Q_1^m, c_1') - \pi^1(c_1', \underline{c}_2)$ represents the greatest possible decrease in profits from the additional entry in the second period. The condition requires that the probability of additional entry is sufficiently small that a deviation is profitable. For distributions with bounded support Proposition 3 is equivalent to the statement that if the variance is "large enough" no complete pooling equilibria exist.

An extreme case is where the noise has unbounded support. For complete pooling we require $\mu^*(c_1) = x = Q_1(c_1)$. But $\mu^*(c_1) = x$ implies that the potential entrant expects all types of established firm to produce the same output. Accordingly, the potential entrant makes no inference from the price itself, but enters if and only if it would have entered based on its prior beliefs. Thus the established firm's actions have no impact on entry. Accordingly each type of established firm would produce its monopoly output. But then $Q_1^*(c_1)$ is strictly decreasing in $c_1$ which contradicts $Q_1^*(c_1) = x$.

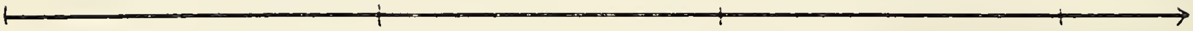
## 3. A MULTIPERIOD MODEL

In a deterministic setting, a two period model is sufficient to capture all the phenomena of interest since all the required signalling can be done by the established firm in the first period. With the introduction of noise, however, only noisy signals can be given in any period and hence, as was seen earlier, only imprecise inferences can be made by the potential entrant. In this context the entrant might have reason to delay entry and, by observing the price in later periods, get a better estimate of $c_1$. To allow for this possibility the model is expanded here to permit multiperiod play.

In extending the model to an n-period setting, the structure of the previous section is maintained as far as possible. The monopolist chooses an output level in each period so long as entry has not occurred. This choice stochastically determines the observed industry price. Armed with this and all previous price realizations, the potential entrant decides whether or not to enter. Once it enters, the firms play out some cooperative or noncooperative duopoly solution forever after.

We use the following timing of events. At time t=0, the established firm makes its choice of period one output, $Q_1$. As before, this gives rise to a first period price $P_1(Q_1,\tilde{\varepsilon})$. We assume that the profit earned in this period accrues at time t=1. Also at time t=1, the potential entrant makes its entry choice $r_1 \varepsilon \{0,1\}$, where $r_1$ indicates that the potential entrant enters and plays its duopoly action. If the potential entrant chooses not to enter, the established firm makes its period two output choice, $Q_2$, also at t=1. This process continues in the natural way. The timing is illustrated in Figure 1 below assuming that entry occurred at time t=2.

FIGURE 1: AN EXAMPLE OF THE TIME SEQUENCE OF MOVES



| $t = 0$ | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|
| Established firm chooses $Q_0$. | Price $P(Q_1,\varepsilon_1)$ is revealed.<br>$\pi^m(Q_1,\varepsilon_1)$ is accrued.<br>Entrant chooses $r_1 = 0$.<br>Established firm chooses $Q_1$. | Price $P(Q_2,\varepsilon_2)$ is revealed.<br>$\pi^m(Q_2,\varepsilon_2)$ is accrued.<br>Entrant chooses $r_2 = 1$. | Established firm and entrant earn $\pi^1(c_1,c_2)$ and $\pi^2(c_1,c_2)$ respectively from here on. |

At any intermediate stage in the evolution of the industry prior to entry, both the information that the players have about each other, as well as the actions that they will find it optimal to take, will depend on the events that have taken place so far. This includes not only the actions that the players have taken in the past, but also (for the established firm) the realizations of the random component of demand each period. As in the model presented earlier, the information possessed by the two players differs. In addition to knowing the price realization each period, the established firm also knows the choice of output whereas the potential entrant only observes the price realizations. The information set of the established firm at time t can be represented as $H_t = \{c_1, Q_1,\ldots,Q_{t-1},\tilde{\varepsilon}_1,\ldots,\tilde{\varepsilon}_{t-1}\}$ at any pre-entry stage. The information set of the potential entrant can be represented as $H_t' = \{c_2,P_1,\ldots,P_{t-1}\}$.

As before, the way in which the entrant evaluates the situation at any point will depend on its beliefs as to the actions each type of established firm has taken. In the two period model these beliefs were described by a function $\mu(c_1):[\underline{c}_1,\overline{c}_1] \to [0,\overline{Q}]$. In the multiperiod model the entrant will have a set of beliefs as to the established firm's actions each period, and realistically these beliefs should be expected to vary with each price-realization. Thus the entrant's beliefs will be described by:[5]

$$\vec{\mu}^{n-1} = \{\mu_1(c_1),\mu_2(c_1,P_1),\ldots\mu_{n-1}(c_1,P_1,P_2,\ldots,P_{n-1})\}.$$

As before we shall concentrate on the case where $\mu_t(c_1,P_1,\ldots)$ is monotonically decreasing in $c_1$ for all t.

In evaluating its situation at any pre-entry stage, the established firm will also make use of all its information. In particular, it will update its beliefs as to the type of potential entrant it is facing given that the potential entrant chose not to enter after having observed the past realized prices. Thus the situation involves a complex two-sided uncertainty problem analogous to that in the model of predation analyzed by Kreps and Wilson in [8].

A complete specification of a pure strategy for each player must stipulate what action it is to take at each date for every possible information set. A sequential equilibrium requires the following:

(i) the established firm's (potential entrant's) strategy must be optimal given its beliefs about the strategy the potential entrant (established firm) is using and its beliefs over what type of entrant (established firm) it is facing given the history of events, $H_t(H_t')$.

(ii) The way in which the established firm (potential entrant) updates its beliefs about the type of entrant (established firm) that it is facing must be consistent with the history of events $H_t(H_t')$ and to the extent possible, with the assumption that the other firm is playing the specified strategy.

(iii) The beliefs that each firm has about the strategy the other is using must in fact be correct.

The notion of payoff used is the discounted present value of expected profits with discount factor $0 \leq \delta < 1$. Thus if the actual types of the established firm and entrant are $c_1'$ and $c_2'$ respectively, and if entry occurs at time t=e, the payoff to the established firm evaluated at t=0 is

$$\pi_e^1(c_1',c_2') = \sum_{t=1}^{e} \delta^t \pi^m(c_1',Q_t) + \sum_{t=e+1}^{n} \delta^t \pi^1(c_1',c_2')$$

and the payoff to the entrant evaluated at t=0 is

$$\pi_e^2(c_1',c_2') = \sum_{t=e+1}^{n} \delta^t \pi^2(c_1',c_2') - \delta^e K.$$

As before, let us examine the entrant's problem first. It faces a more complicated problem in the multiperiod case than in the two period problem. Not only must the potential entrant evaluate whether its expected profits are positive, but also, if they are, whether it is worth delaying entry in order to gather more precise information as to the existing firm's type from additional realized prices.

Thus a potential entrant that has not yet entered enters at time e if and only if:

(i) $E_e[\pi_e^2(c_1,c_2) \mid H_e'] \geq 0$, and

(ii) $E_e[\pi_e^2(c_1,c_2) \mid H_e'] \geq E_e\delta^{\ell-e}[\text{Max}\ \{E_\ell[\pi_\ell^2(c_1,c_2) \mid H_e'],0\}$ for all $\ell$, $n \geq \ell > e$.

The first condition requires that expected profits be nonnegative. The second requires that the discounted expected profits from waiting to a later date be no greater than from entering immediately. The subscript on the expectation indicates the date at which the expectation is taken. Thus in the right-hand-side of (ii) the subscript "$\ell$" on the expectation indicates that the expectation is conditioned not only on the prices $\{P'_1,\ldots,P'_e\}$ but also $\{P'_{e+1},\ldots P'_\ell\}$. Some realizations of $\{P'_{e+1},\ldots P'_\ell\}$ will be such that $E_1[\pi^2(c_1,c_2) \mid P_1',\ldots P_\ell]$ is positive in which case the potential entrant will choose to enter. In the cses where this quantity is negative the potential entrant will choose to stay out. The expected profit if the potential entrant waits until time $\ell$ thus depends on the relative probability (evaluated at time e) of the possible sequences of $\{P'_{e+1},\ldots,P'_\ell\}$. This is captured by the subscript "e" on the outer expectation.

If the potential entrant enters at time e its expected profits (evaluated at time 0) are:

$$E_e[\pi_e^2(c_1,c_2) \mid H'_e] =$$

$$\int_{\underline{c}_1}^{\overline{c}_1} \sum_{t=e+1}^{n} \delta^t \pi^2(c_1^*,c_2) Pr(c_1=c_1^* \mid H'_e) dc_1^* - \delta^e K.$$

Let the potential entrant's expected profits, evaluated at time e, from not entering at time e be $E_e\pi_e^2(c_1,c_2 \mid r_e=0,H'_e)$. Then

$$E_e\pi_e^2(c_1,c_2 \mid r_e=0,H'_e) =$$

$$E_e\{Max[E_{e+1}[\pi_{e+1}^2(c_1,c_2) \mid H'_{e+1}],[E_{e+1}\pi_{e+1}^2(c_1,c_2 \mid r_{e+1}=0) \mid H'_{e+1}]]\}$$

where the expectation "e" is taken over the possible values of $P'_{e+1}$ contained in $H'_{e+1}$.

The tradeoff facing the entrant in evaluating whether to enter at e or wait to e+1 is whether it is worthwhile foregoing an expected profit in the forthcoming period for an additional price-realization $P'_{e+1}$. The value in receiving $P'_{e+1}$ comes from the flexibility of being able to stay out of a market that becomes *ex post* unprofitable after receiving the additional information. Let $E[G_{e,e+1}]$ be the change in the expected payoff from the next period when the entry decision is delayed by one period (evaluate at time t=0). As argued above, and shown in Lemma 5, if entry is profitable at time e the value of the additional information is nonnegative indicating the possibility that the potential entrant will choose to delay entry.

LEMMA 5: $E[G_{e,e+1}] > 0$.

PROOF:

$$E[G_{e,e+1}] = E_e[\pi_e^2(c_1,c_2 \mid r_e=0,H_e')]-[\int_{\underline{c}_1}^{\overline{c}_1} \sum_{t=e+2}^{n} \delta^t\pi^2(c_1^* c_2 \mid H_e')dc_1^* - \delta^e K]$$

$$\geq E_e[E_{e+1}[\pi_{e+1}^2(c_1,c_2) \mid H_{e+1}']] - [\int_{\underline{c}_1}^{\overline{c}_1} \sum_{t=e+2}^{n} \delta^t\pi^2(c_1,^* c_2 \mid H_e')dc_1^* - \delta^e K]$$

$$= E_e[\int_{\underline{c}_1}^{\overline{c}_1} \sum_{t=e+2}^{n} \delta^t\pi^2(c_1^*,c_2 \mid H_{e+1}')dc^*_1] - \delta^{e+1}K$$

$$- [\int_{\underline{c}_1}^{\overline{c}_1} \sum_{t=e+2}^{n} \delta^t\pi^2(c_1^*,c_2 \mid H_e')dc^*_1 - \delta^e K]$$

$= \delta^e K - \delta^{e+1}K > 0$ by a well-known result in probability theory. Q.E.D.

Given the possibility that the entrant will choose to delay entry in order to gather more information, the question arises as the whether the optimal strategy for the entrant will still be to use a trigger-price. For this to be so, it must be the case that:

(Condition 1): Expected profits are monotonically increasing in the current period price, and that

(Condition 2): If it isn't worthwhile for the potential entrant to delay entry when it observes a relatively low current period price, then it isn't worthwhile to delay entry if it observes a relatively high current period price.

From the arguments used in the proof of Lemma 2 it is clear that the first condition always holds. The combination of the fact that the entrant expects low-cost established firms to produce more than high-cost established firms and the fact that the monotone likelihood ratio property holds, guarantees that high prices are more likely to be associated with a high-cost established firm than a low-cost one. These facts alone are insufficient, however, to guarantee that the second condition holds. This is because the entrant's beliefs about the following period's actions may depend on past prices. For example, suppose the entrant believes that the established firms of all types produce very similar outputs when the previous price was high, but that they produce widely differing outputs if the previous price was low. Then the potential entrant will believe it will learn a great deal about the established firm's type from the next price-realization if and only if the previous price was low. If its expected profits are positive it may therefore choose to delay entry if it observes the low price but to enter immediately if it observes the high price.

Somewhat surprisingly, a weak set of conditions are sufficient to rule out this type of behavior and to guarantee that Condition 2 holds and hence that trigger-pricing is the optimal entrant strategy even in a multiperiod model. The axiom in the two period model that posits that the entrant believes that the high-cost established firms produce less than the low-cost established firms is replaced by an axiom that requires that this monotonicity holds every period. Let the vector of the established firm's outputs be $\vec{Q}^t(c_1) = (Q_1,\ldots,Q_t)$.

PROPOSITION 4: The following conditions are sufficient to guarantee that a trigger-price strategy is optimal for the potential entrant in a multiperiod model:

(a) $\{f(\vec{P}^t \mid \vec{Q}^t)\}$ have the monotone likelihood ratio property for all t

(b) $\vec{\mu}^t(c'_1) < \vec{\mu}^t(c''_1)$ if and only if $c'_1 \geq c''_1$ for all t.

PROOF:[6] Combining Lemmas 6 and 7 (see the appendix), if (a) and (b) hold then $P'_1 > P''_1 \rightarrow E[G_{1,2} \mid P'_1] \leq E[G_{1,2} \mid P''_1]$. Thus the payoff to delay is monotonically decreasing in the current period price and hence Condition 2 holds. Condition 1 follows directly from (a) and (b) by the proof of Lemma 2. Q.E.D.

If the entrant uses a trigger-price strategy, the structure of any stage game is similar to the one-shot game studied in Section 2. Entry is more probable at any stage the higher the realized price, and hence the same incentive to limit-price exists.

PROPOSITION 5: An equilibrium of the multiperiod game involves limit-pricing at each pre-entry stage.

PROOF: Let $\pi_t^*(Q_t,c_1)$ be the maximum profit the established firm can earn (evaluated at time e). Then

$$\pi_t^*(Q_t,c_1) = \max_{Q_t} \delta\pi^m(Q_t,c_1) + \int_{\underline{c}_2}^{\bar{c}_2} \Big[ \sum_{t=2}^{n-e} \delta^t \pi^1(c_1,c_2^*)(1-F_t(P_t^*(c_2^*) \mid Q_t)$$

$$+ \delta^2 \pi^m(Q_{t+1}, c_1)F_t(P_t^*(c_2^*) \mid Q_t)$$

$$+ \delta^2 \pi_{t+1}^*(Q_t,Q_{t+1},c_1)F_t(P_t^*(c_2^*) \mid Q_t)\Big] h_1(c_2^*)dc_2^*.$$

Rewriting, this is equal to

$$\max_{Q_t} \delta\pi^m(Q_t,c_1) - \int_{\underline{c}_2}^{\bar{c}_2} [R](1-F_t(P_t^*(c_2^*) \mid Q_t)h_1^*(c_2^*)dc_2^*$$

$$+ \delta^2\pi^m(Q_{t+1},c_1) + \delta^2\pi_{t+1}(Q_t,Q_{t+1},c_1)$$

where

$$R = \delta^2\pi^m(Q_{t+1},c_1) + \delta^2\pi_{t+1}^*(Q_t,Q_{t+1},c_1) - \sum_{t=2}^{n-e} \delta^t\pi^1(c_1,c_2)$$

is the reward to deterring entry for a period. Evaluating the derivative of the t-th period profit at the optimum and rearranging gives:

$$\left.\frac{\partial \pi^m(Q_t, c_1)}{\partial Q_t}\right|_{Q_t^*}$$

$$= \int [R] f_t(P_t^*(c_2^*) \mid Q_t) h_1(c_2^*) dc_2^* \left.\frac{\partial P_t(Q_t, \tilde{\varepsilon})}{\partial Q_t}\right|_{Q_t^*} - \delta \left.\frac{\partial \pi_{t+1}^*(Q_t, Q_{t+1}, c_1)}{\partial Q_t}\right|_{Q_t^*}$$

$$< 0$$

since the reward to deterring entry is positive and

$$\left.\frac{\partial \pi_{t+1}^*(Q_t, Q_{t+1}, c_1)}{\partial Q_t}\right|_{Q_t = Q_1^*} > 0$$

because an increase in current period output results in a lower current period price, in expectation, which makes present and future entry less attractive and hence raises future profits. Q.E.D.

In equilibrium the entrant uses the correct inference structure about the established firm's costs. Therefore as $n \to \infty$, by waiting long enough, the sample of observed prices grows arbitrarily large and the entrant can estimate the costs of the established firm to any degree of precision. If n is "large enough", therefore, a potential entrant that stays out of the market will ultimately have almost complete information (and in the limit as $n \to \infty$ will have complete information). In this case any type of potential entrant that would have entered ex ante with complete information, will ultimately enter here as well. In addition, in early period some entrants that would not have entered with perfect information, will, in expectation, mistakenly enter because of stochastically high realized prices. Therefore, there is unambiguously more expected entry than with complete information or in a separating equilibrium with incomplete information but no exogenous noise.

PROPOSITION 6: There exists a T such that for all $c'_1$

$$\sum_{t=1}^{T} \Pr\{r_t=1 \mid c'_1\} > \Pr\ \{c_2 \leq c'_2 \equiv \inf_{c_2} ( \sum_{t=1}^{T} \delta^t \pi^2(c'_1,c_2) - K \geq 0)\}.$$

PROOF:

$$\Pr\{r_t=1 \mid c'_1\} = \int_{\underline{c}_2}^{\bar{c}_2} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^*.$$

Therefore

$$\sum_{t=1}^{T} \Pr\{r_t=1 \mid c'_1\} = \int_{\underline{c}_2}^{\bar{c}_2} \sum_{t=1}^{T} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^*.$$

Now for all $\eta > 0$ there exists a $T(c_2^*,\eta)$ such that

$$\sum_{t=1}^{T} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} > 1 - \eta \text{ if } c_2^* < c'_2$$

where $c'_2$ is the type of entrant that would be indifferent between entering and staying out if it had complete information by the argument given above (since the entrant is making the appropriate inferences).

$$\text{Let } T(\eta) = \sup_{c_2^*} T(c_2^*,\eta).$$

Now for all $\eta > 0$

$$\int_{\underline{c}_2}^{\bar{c}_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^*.$$

$$= \int_{\underline{c}_2}^{c'_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^* + \int_{c'_2}^{\bar{c}_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^*$$

$$> (1-\eta) \Pr\{c_2 \leq c'_2\} + \int_{c'_2}^{\bar{c}_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* \mid c'_1\} h_2(c_2^*) dc_2^*.$$

Now

$$\int_{c_2'}^{\bar{c}_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* | c_1'\} h_2(c_2^*) dc_2^* \geq \int_{c_2'}^{\bar{c}_2} \sum_{t=1}^{T(\eta)} \Pr\{r_t=1, c_2=c_2^* | c_1'\} h_2(c_2^*) dc_2^* >$$

$$\gamma \text{ for some } \gamma > 0;$$

i.e., there is positive expected mistaken entry and it is at least as great as the positive expected mistaken entry in the first period. So choose T so that $\eta < \gamma/\Pr\{c_2 \leq c_2'\}$. Q.E.D.

## 4. DISCUSSION

A major characteristic of a many-period model is that the existence of exogenous noise may result in delayed entry. Clearly this has negative welfare implications for the consumers since it results in their being faced with monopoly for an extended period. On the other hand the noise also induces some mistaken entry which will typically result in depressed prices with the accompanying prositive welfare implications. Consumers will also benefit from the low pre-entry prices that arise from the established firm's attempt to demonstrate, as it must in equilibrium, that it has a relatively low cost structure and is therefore not an attractive candidate with which to share an industry. Which of these opposing effects will dominate will depend on the value of the parameters.

A number of important facets of the competitive structure have not been captured here. For example, the presence of an experience curve phenomenon would increase the cost to the potential entrant of delaying entry while at the same time increasing the benefit to the established firm of causing delayed entry. In view of the work of Spence[16] this is likely to result in even greater pre-entry output.

The subgame that is to be played after entry has occurred has not been modeled here. In fact, once entry has occurred the presence of incomplete information makes predaton of the kind analyzed by Kreps and Wilson[8] and Milgrom and Roberts[11] a real possibility. Even without predation the problem of how the noncooperative post-entry game with noise would be played remains.

Finally, there is only one potential entrant in this model. The model could be greatly enriched by the introduction of multiple entrants. In this case rival entrants have to be concerned not only with the type of established firm in the industry, but also with the type of the other potential entrants and whether the industry would remain profitable if additional entry occurred.

## 5. SUMMARY AND CONCLUSIONS

This paper has presented a two period and multiperiod model of the behavior of an established firm when faced with potential entry. A key feature of the models is that lack of accurate information possessed by the players. Not only is each firm unsure as to the precise type (costs) of its opponent, but in addition they operate in an environment where, because of exogenous demand uncertainty, the entrant is unsure of the precise actions begin taken by the established firm. Provided the structure of the exogenous demand uncertainty is such that prices are more likely to be associated with low outputs than high outputs, the optimal strategy of the entrant will be to enter if and only if the price exceeds some critical level. Thus the probability of entry is an increasing function of price as assumed in earlier dynamic models. This behavior on the part of the entrant induces the established firm to expand its output beyond the monopoly level in order to reduce the probability of entry.

In a multiperiod model entry may be delayed rather than deterred as the potential entrant decides to observe more price-realizations before committing itself to entry. If the time horizon is long enough, there will ultimately be more entry than with perfect information. This results from the fact that eventually the entrant gains a precise estimate of the type of established firm occupying the industry and hence enters if it would with perfect information. In addition a potential entrant that could not enter with perfect information may be induced to enter by a stochastically high price.

# APPENDIX

PROPOSITION 3: If for all $x \geq Q_1^m(\underline{c}_1)$

$$1 - F(\overline{P}_1 \mid y) < \frac{\pi^m(y, c_1') - \pi^m(x, c_1')}{\pi^m(Q_1^m, c_1') - \pi^1(c_1', \underline{c}_2)}$$

for some $y < Q_1^m(\underline{c}'_1)$ and some $c_1' \in [\underline{c}_1, \overline{c}_1]$, then no complete pooling equilibrium exists.

PROOF: Notice firstly that there can be no complete pooling equilibrium at an output $x < Q_1^m(\underline{c}_1)$ since an established firm of type $\underline{c}_1$ would want to deviate to its monopoly level. This follows since by moving to $Q_1^m(\underline{c}_1)$ it both increases its pre-entry profits and, by the monotonicity of entrant beliefts, results in less entry.

Suppose then that there exists a pooling equilibrium at some $x \geq Q_1^m(\underline{c}_1)$. Then an established firm of type $c_1'$ earns

$$\pi^m(x, c_1') + \pi^m(Q_1^m, c_1')\Pr\{c_2' \geq c_2^*\} + \pi^1(c_1', c_2')\Pr\{c_2' < c_2^*\} \tag{A.1}$$

if it produces output x, where $c_2^*$ is the type of entrant that is indifferent between entering and staying out of the industry based only on its priors.

An established firm that deviates to y earns

$$\pi^m(y, c_1') + [\pi^m(c_1')\Pr(c_2' > c_2^*) + \pi^1(c_1', c_2')\Pr(c_2' < c_2^*)]F(\overline{P}_1 \mid y)$$
$$+ [\text{Profits if } P_1' > \overline{P}_1](1 - F(\overline{P}, \mid y)) \tag{A.2}$$

The term $[\text{Profits if } P_1^* > \overline{P}_1]$ will depend on the exact nature of the entrant's beliefs off the equilibrium path. A deviation to y is worthwhile to an established firm of type $c_1'$ if

$$\pi^m(y,c_1') - \pi^m(x,c_1') >$$

$$(1-F(\overline{P}_1 \mid y))[\pi^m(c_1')\Pr(c_2' \geq \overset{*}{c}_2) + \pi^1(c_1',c_2')\Pr(c_2 < \overset{*}{c}_2)-[\text{Profits if } P_1 > \overline{P}_1]].\text{(A.3 )}$$

Since $\pi^m(c'_1)\Pr(c'_2>\overset{*}{c}_2)+\pi^1(c'_1,c'_2)\Pr(c_2<\overset{*}{c}_2)\leqslant\pi^m(c'_1)$

and

$$[\text{Profits if } P_1>\overline{P}_1]>\pi^1(c_1',\underline{c}_2),$$

Equation (A.3) reduces to

$$\Pr\{P_1(y,\varepsilon)>\overline{P}_1\}[\pi^m(c_1') - \pi^1(c_1,\underline{c}_2)] < \pi^m(y,c'_1) - \pi^m(x,c_1').$$

Q.E.D.

LEMMA 3:$m\{c_2' : f(\overset{*}{P}_1(c_2') \mid \overset{*}{Q}_1>0)\} > 0$ for all $\overset{*}{Q}_1$.

PROOF: Recall that $\overline{P}_1(\underline{P}_1)$ is the supremum (infimum) of the expected prices.

Case(i): $\overline{P}_1 = \infty$.

This is the case if and only if the support of $F(P_1 \mid Q_1)$ is unbounded from above. However since $\mu(c_1)$ is strictly monotone, the set of types of entrant that have trigger prices $\infty>\overset{*}{P}_1(c'_2) > 0$ has positive measure. But since $f(P_1 \mid Q_1)$ has unbounded support, $f(\overset{*}{P}_1(c_2') \mid \overset{*}{Q}_1)>0$ for some realization of $\tilde{\varepsilon}$ and all $\overset{*}{Q}_1$, $P_1(\overset{*}{Q}_1,\tilde{\varepsilon}) = \overset{*}{P}_1(c_2')$ for some $\tilde{\varepsilon}$ and all $\overset{*}{Q}_1$.

Case(ii): $\overline{P}_1< \infty$.

$\overline{P}_1 < \infty$ if and only if the support of $f(P_1 \mid Q_1)$is bounded from above. Now consider some $Q_1'\varepsilon[\mu(\underline{c}_1),\mu(\overline{c}_1)]$. Let $[P',P'']$be the range of possible prices given

$Q_1'$. Define

$$c_2'' \equiv \inf_{c_2}\{E[\pi^2(c_1,c_2) \mid P'']-K \le 0\}$$

$$c_2' \equiv \sup_{c_2}\{E[\pi^2(c_1,c_2) \mid P'] -K \ge 0\}.$$

If $c_2' \ne c_2''$ then for all $c_2\ \varepsilon(c_2'',c_2')$ there is a trigger price $P_1^*\ \varepsilon[P',P'']$. Therefore $m\{c_2' : f(P_1^*(c_2') \mid Q_1')>0\}>0$.

If $c_2' = c_2''$ then $E[\pi^2(c_1,c_2) \mid P''] = E\{\pi^2(c_1,c_2) \mid P']$ which implies that there is a neighborhood of $Q_1'$ not produced by any type of established firm. But this implies that $\mu^*(c_1)$ is not continuous in equilibrium which contradicts Proposition 2. Q.E.D.

LEMMA 6: If Conditions 1 and 2 hold then

$$P_1' > P_1'' \rightarrow E_2[\pi_2^2(c_1,c_2) \mid P_1',P_2'] \ge E_2[\pi_2^2(c_1,c_2) \mid P_1'',P_2'].$$

PROOF: By the proof of Lemma 1, if Conditions 1 and 2 hold then $\vec{P}^{2'} > \vec{P}^{2''} \rightarrow$

$$E_2[\pi_2^2(c_1,c_2) \mid \vec{P}^{2'}] \ge E_2[\pi_2^2(c_1,c_2) \mid \vec{P}^{2''}].$$

Since $P_1' > P_1''$ implies $\vec{P}^{2'} = (P_1',P_2') > (P_1'',P_2') = \vec{P}^{2''}$, the result follows immediately. Q.E.D.

LEMMA 7: If $P_1' > P_1'' \rightarrow E_2\{\pi_2^2(c_1,c_2) \mid P_1',P_2'] \ge E_2\{\pi_2^2(c_1,c_2) \mid P_1'',P_2']$ then $P_1' > P_1''$ $\rightarrow E[G_{1,2} \mid P_1'] \le E[G_{1,2} \mid P_1'']$.

PROOF: $E[G_{1,2} \mid P_1'] = \delta E_{P_2} \mathrm{Max}\{E_2[\pi_2^2(c_1,c_2) \mid P_1',P_2],0\} - \delta E_2[\pi_2^2(c_1,c_2) \mid P_1']$ where the subscript on the expectation indictes that the expectation is over the possible prices $P_2$ i.e. it is the expectation taken at time 1. Therefore

$$E\{G_{1,2} \mid P_1''] - E\{G_{1,2} \mid P_1'] = \delta E_{P_2 > P_2''} E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] - \delta E_2[\pi_2^2(c_1,c_2) \mid P_1'']$$

$$- [\delta E_{P_2 > P_2'} E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2] - \delta E_2[\pi_2^2(c_1,c_2) \mid P_1']]$$

where $P_2''$ and $P_2'$ are defined by

$$E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2''] = 0$$

$$E_2(\pi_2^2(c_1, c_2) \mid P_1', P_2'] = 0.$$

Rearranging gives

$$-\delta[E_{P_2 < P_2''} E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] - E_{P_2 < P_2'} E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2] \geq 0$$

as follows: $E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] \leq E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2]$ if $P_1' > P_1''$

$$\rightarrow 0 = E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2'] > E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2]$$

$$\rightarrow E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2'] < E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2''] = 0$$

$$\rightarrow P_2' < P_2''$$

since $E_2[\pi_2^2(c_1,c_2) \mid P_1, P_2]$ is monotone decreasing in $P_2$. Therefore

$$E_{P_2 < P_2''} E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] - E_{P_2 < P_2'} E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2] =$$

$$E_{P_2 < P_2'}[E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] - E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2]] + E_{P_2' < P_2 < P_2''} E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2]$$

$$\leq 0$$

since $E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] < E_2[\pi_2^2(c_1,c_2) \mid P_1', P_2]$ for all $P_2$ and

$E_2[\pi_2^2(c_1,c_2) \mid P_1'', P_2] < 0$ for all $P_2 < P_2''$. Q.E.D.

# FOOTNOTES

[1] See F. Scherer [15] and K.O. Nti and M. Shubik [13] for references to this class of papers. The maintained output hypothesis is discussed in [15:pp 232-252].

[2] See, for example, Friedman [3,4], Gal-Or [5], Matthews and Mirman [12], Milgrom and Roberts [10] and Rogerson [14].

[3] See Milgrom [9] for an excellent discussion of the monotone likelihood ratio property and related representation theorems.

[4] For example, Scherer [15] writes: "... unless an industry enjoys substantial barriers to new entry, a higher price and profit margin policy could attract new entrants, whose additional output will have a depressing impact on future prices and profits".

[5] The arrow indicates vector notation while the superscript n-1 denotes the last element in the vector.

[6] The proof is present for the three period model. The extension to the general n-period model is straighforward and is not presented here.

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